

Precalculus Notes: Solving Trig Equations

Use the restricted domain and inverse function to solve the trig equations. Across which quadrants will you find the solutions for..

sine and cosecant? $\frac{1, 4}{\text{---}}$
 tangent? $\frac{1, 4}{\text{---}}$

cosine and secant? $\frac{1, 2}{\text{---}}$
 cotangent? $\frac{1, 2}{\text{---}}$

Examples: Solving linear and quadratic forms

EX 1: $4\sin x = 2\sin x + \sqrt{2}$ **LINEAR**

$$2\sin x = \sqrt{2}$$

$$\sin x = \frac{\sqrt{2}}{2}$$

$$x = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$$

$$x = 45^\circ \text{ or } \frac{\pi}{4}$$

EX 2: $\cos\theta\sin\theta = 3\cos\theta$ **GCF**

GCF $\cos\theta\sin\theta - 3\cos\theta = 0$

$$\cos\theta(\sin\theta - 3) = 0 \quad \text{2PP}$$

$$\cos\theta = 0 \quad \sin\theta - 3 = 0$$

$$\theta = \cos^{-1}(0) \quad \sin\theta = 3$$

$$\theta = \pi/2 \text{ or } 90^\circ \quad \sin^{-1}(3) = \text{UND}$$

EX 3: $6\tan^2 x - 2 = 4$ **Square Rooting**

$$6\tan^2 x = 6$$

$$\tan^2 x = 1$$

$$\tan x = \pm 1$$

$$\tan x = 1 \quad \tan x = -1$$

$$\tan^{-1}(1) = \frac{\pi}{4} \quad \tan^{-1}(-1) = -\frac{\pi}{4}$$

EX 4: $2\sin^2\theta = \sin\theta + 1$ **Big X**

$$2\sin^2\theta - \sin\theta - 1 = 0$$

$$\sin\theta = 1 \quad \sin\theta = -\frac{1}{2}$$

$$\sin^{-1}(1) = \frac{\pi}{2}$$

$$\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

(Note: A quadratic formula diagram is shown with a large 'X' over it, indicating it was not used.)

You try: $3\tan x - \sqrt{3} = 0$

$$3\tan x = \sqrt{3}$$

$$\tan x = \frac{\sqrt{3}}{3}$$

$$x = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

You try: $2\sin\theta\cos\theta = 2\cos\theta$

$$2\sin\theta\cos\theta - 2\cos\theta = 0$$

$$2\cos\theta(\sin\theta - 1) = 0$$

$$2\cos\theta = 0 \quad \sin\theta - 1 = 0$$

$$\cos\theta = 0 \quad \cos^{-1}(0) = \frac{\pi}{2}, \sin^{-1}(1) = \frac{\pi}{2}$$

You try: $6\cos^2\theta - 2 = 1$

$$6\cos^2\theta = 3$$

$$\cos^2\theta = \frac{1}{2}$$

$$\cos\theta = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$$

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \frac{3\pi}{4}$$

You try: $4\sin^2\theta - 4\sin\theta + 1 = 0$ **mult 2**

$$\sin\theta = \frac{1}{2}$$

$$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \text{ (mult 2)}$$

(Note: A quadratic formula diagram is shown with a large 'X' over it, indicating it was not used.)

Steps:

1. Isolate the trig function.
2. Write the reciprocal equation.
3. Take the inverse of the reciprocal.

**** Special Note: When using a calculator to find the inverse of *cotangent*, there are two special circumstances:

- If you get a negative value you must add π or 180° to each solution
- If you get 0, then the value will be equal to $\frac{\pi}{2}$.

<p>EX 1: $5 = \csc^2 x + 3$</p> $2 = \csc^2 x$ $\frac{1}{2} = \sin^2 x$ $\pm \sqrt{\frac{1}{2}} = \sin x$ $\pm \sqrt{\frac{2}{2}} = \sin x$	<p>EX 2: $\sec^2 \theta - \sec \theta = 2$</p> $\sec^2 \theta - \sec \theta - 2 = 0$ <p>$\sec \theta = +2, \sec \theta = -1$</p> <p>$\cos \theta = \frac{1}{2}, \cos \theta = -1$</p> <p>$\cos^{-1}(\frac{1}{2}) = 60^\circ, \cos^{-1}(-1) = 180^\circ$</p>
<p>EX 3: $\csc^2 \theta + 2 \csc \theta = 0$</p> $\csc \theta (\csc \theta + 2) = 0$ <p>$\csc \theta = 0$ $\csc \theta + 2 = 0$</p> <p>$\frac{1}{\sin \theta} = 0$ $\csc \theta = -2$</p> <p>where $\sin \theta$ is undefined \rightarrow NO SOLUTIONS</p> <p>$\sin \theta = -\frac{1}{2}$</p> <p>$\sin^{-1}(-\frac{1}{2}) = -30^\circ$</p>	<p>EX 4: $-1 + \cot^2 \theta = 0$</p> $\cot^2 \theta = 1$ <p>$\cot \theta = \pm 1$</p> <p>$\tan \theta = \pm 1$</p> <p>$\tan^{-1}(1) = 45^\circ$ $\tan^{-1}(-1) = -45^\circ$</p> <p>$+180^\circ$</p> <p>$\rightarrow 135^\circ$</p>
<p>EX 5</p>	<p>You try:</p> <p>\rightarrow You must add 180° to account for domain restriction</p>

Trig. Equations Worksheet 1

Solve for θ in the interval: $0 \leq \theta < 360^\circ$

DEGREES

1. $\cos \theta + 1 = 0$

$\cos \theta = -1$

$\cos^{-1}(-1) = \boxed{180^\circ}$

2. $\sin^2 \theta = 0$

$\sin \theta = \pm 0$

$\sin^{-1}(0) = \boxed{90^\circ}$

3. $2 \cos \theta - \sqrt{3} = 0$

$2 \cos \theta = \sqrt{3}$

$\cos \theta = \frac{\sqrt{3}}{2}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{30^\circ}$

4. $\cos \theta (\tan^2 \theta - 3) = 0$

$\cos \theta = 0$ $\tan^2 \theta - 3 = 0$

$\cos^{-1}(0) = \boxed{90^\circ}$ $\tan^2 \theta = 3$

$\tan \theta = \pm \sqrt{3}$

Solve for ALL values of θ using degrees

$\tan^{-1}(\sqrt{3}) = \boxed{60^\circ}$ $\tan^{-1}(-\sqrt{3}) = \boxed{-60^\circ}$

5. $2 \cos^2 \theta = 3 \cos \theta - 1$

$2 \cos^2 \theta - 3 \cos \theta + 1 = 0$

$\cos \theta = 1$

$\cos \theta = \frac{1}{2}$

$\frac{-2 \pm \sqrt{2^2 - 4(-3)(1)}}{2(-3)}$

$\cos^{-1}(1) = 0^\circ$
 $\cos^{-1}\left(\frac{1}{2}\right) = \boxed{60^\circ}$

6. $2 \cos^2 \theta - \cos \theta = 1$

$2 \cos^2 \theta - \cos \theta - 1 = 0$

$\cos \theta = 1$

$\cos \theta = -\frac{1}{2}$

$\cos^{-1}(1) = 0^\circ$
 $\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{120^\circ}$

7. $3 \tan^2 \theta - 1 = 0$

$3 \tan^2 \theta = 1$

$\tan^2 \theta = \frac{1}{3}$

$\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \boxed{30^\circ}$
 $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \boxed{-30^\circ}$ solve for θ in $[0, 2\pi)$

8. $4 \cos^2 \theta - 1 = 2$

$4 \cos^2 \theta = 3$

$\cos^2 \theta = \frac{3}{4}$

$\cos \theta = \pm \sqrt{\frac{3}{4}} = \pm \frac{\sqrt{3}}{2}$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{30^\circ}$ $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \boxed{150^\circ}$

$6 \sin^2 \theta = 3$

$\sin^2 \theta = \frac{1}{2}$

$\sin \theta = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{45^\circ}$

$\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-45^\circ}$

RADIANS

10. $2 \cos \theta + 4 = 5$

$2 \cos \theta = 1$

$\cos \theta = \frac{1}{2}$

$\cos^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{3}}$

11. $5 \sin \theta - \sqrt{3} = 3 \sin \theta$

$2 \sin \theta - \sqrt{3} = 0$

$2 \sin \theta = \sqrt{3}$

$\sin \theta = \frac{\sqrt{3}}{2}$

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3}}$

12. $4 \sin^2 \theta - 2 = 0$

$4 \sin^2 \theta = 2$

$\sin^2 \theta = \frac{1}{2}$

$\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$
 $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{-\frac{\pi}{4}}$ $\sin \theta = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$

13. $6 \sin^2 \theta = 5 \sin \theta + 4$

$6 \sin^2 \theta - 5 \sin \theta - 4 = 0$

$\sin \theta = \frac{4}{3}$

$\sin \theta = -\frac{1}{2}$

$\frac{-(-5) \pm \sqrt{(-5)^2 - 4(6)(-4)}}{2(6)}$
 $\frac{5 \pm \sqrt{25 + 96}}{12}$
 $\frac{5 \pm \sqrt{121}}{12}$
 $\frac{5 \pm 11}{12}$
 $\frac{16}{12} = \frac{4}{3}$ $\frac{-6}{12} = -\frac{1}{2}$

14. $\sin^2 \theta - 4 \sin \theta - 5 = 0$

$\sin \theta = 5$

$\sin \theta = -1$

$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-5)}}{2(1)}$
 $\frac{4 \pm \sqrt{16 + 20}}{2}$
 $\frac{4 \pm \sqrt{36}}{2}$
 $\frac{4 \pm 6}{2}$
 $\frac{10}{2} = 5$ $\frac{-2}{2} = -1$

$\sin^{-1}(5) = \text{UND}$

$\sin^{-1}(-1) = \boxed{-\frac{\pi}{2}}$

15. $16 \cos^2 \theta - 8 = 0$

$16 \cos^2 \theta = 8$

$\cos^2 \theta = \frac{1}{2}$

$\cos \theta = \pm \sqrt{\frac{1}{2}} = \pm \frac{\sqrt{2}}{2}$

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\frac{\pi}{4}}$

$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4}}$

$\sin^{-1}\left(\frac{4}{3}\right) = \text{UND}$

$\sin^{-1}\left(-\frac{1}{2}\right) = \boxed{-\frac{\pi}{6}}$

