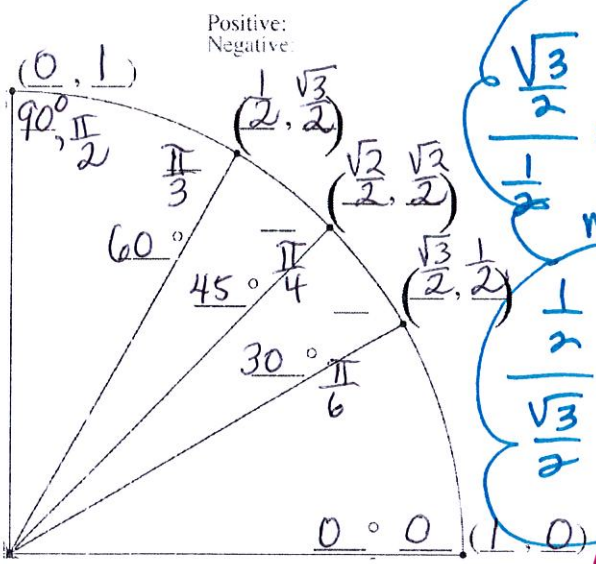
Important Things to Know About the Unit Circle

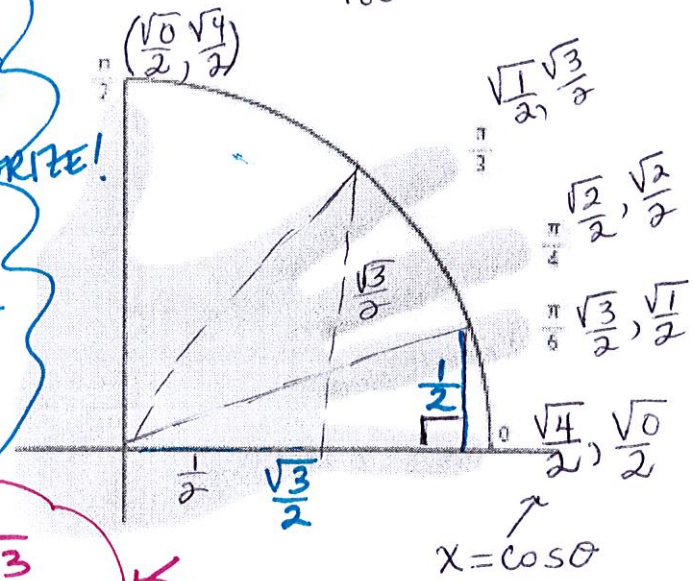
1. You only need to memorize Quadrant I of the unit circle. The other three quadrants can be completed using reflections across the vertical and horizontal axes. You may use the "hand trick" to help memorize the ordered pairs.

$30° \times \frac{\pi}{180°} = \frac{30°\pi}{180°}$



$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$ MEMORIZE!

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$



$\frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ MEMORIZE!

2. Remember that the unit circle is constructed from "special right" triangles. All short leg measures are $\frac{1}{2}$, medium leg measures are $\sqrt{2}/2$, and long leg measures are $\sqrt{3}/2$.

3. On the unit circle, the ordered pairs, (x, y), represent (cos, sin). The tangent function can be evaluated using $\frac{y}{x}$ or $\frac{\sin}{\cos}$. The **reciprocal** functions are

$\cos\theta = \frac{adj}{hyp}$ | $\boxed{\sec\theta} = \frac{1}{\cos\theta} = \frac{hyp}{adj}$ secant | $\boxed{\csc\theta} = \frac{1}{\sin\theta} = \frac{hyp}{opp}$ cosecant

4. There are a finite number of values when evaluating trig functions using the unit circle. Let's complete the table for Quadrant I:

θ (deg/rad)	cos θ	sin θ	tan θ	sec θ	csc θ	cot θ
0	1/1	0/1	0/1 = 0	1	1/0 = UND	1/0 = UND
30	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	* $\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{3}$	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	2	* $\frac{\sqrt{3}}{2} \div \frac{1}{2} = \sqrt{3}$
45	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
60	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	2	$\frac{2\sqrt{3}}{3}$	$\sqrt{3}/3$
90	0/1	1	1/0 = UND	1/0 UND	1	0/1 = 0

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \boxed{\frac{\sqrt{3}}{3}}$ *

Cotangent $= \frac{1}{\tan} = \frac{adj}{opp} = \frac{\cos}{\sin}$

$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} \rightarrow \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

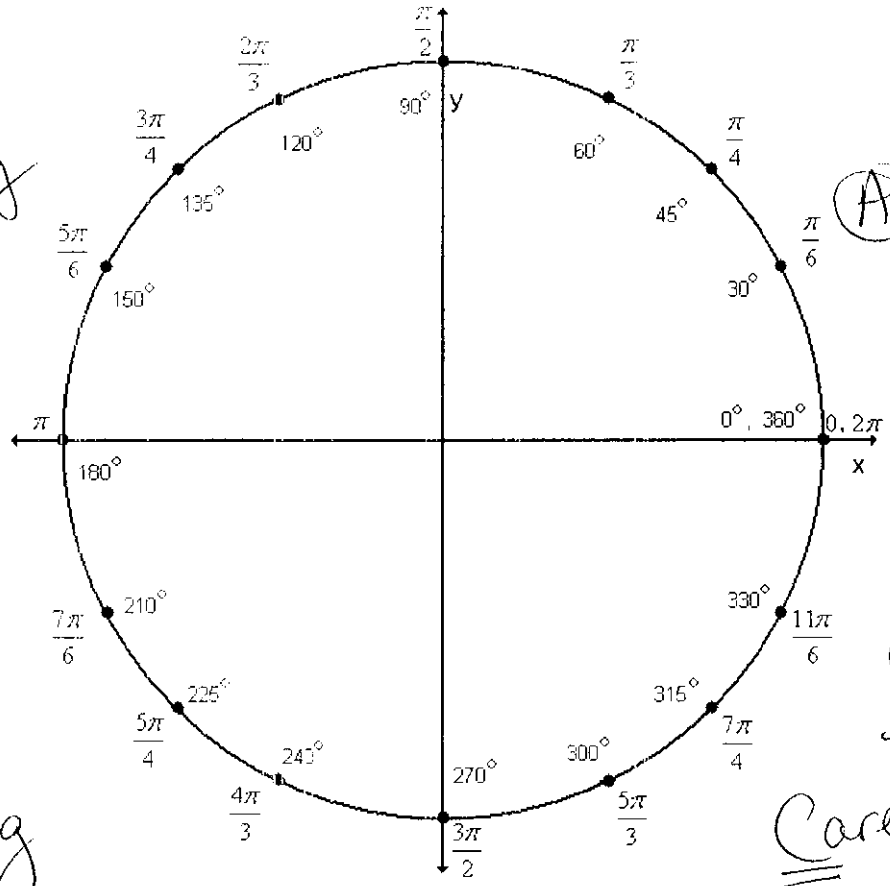
UNIT 6 WORKSHEET 4

USING THE UNIT CIRCLE

Study

Always

sin, csc +



Cos, + Sec

Carefully

Trig

Use the unit circle above to find the exact value of the six trigonometric functions for each of the following angles.

A) $\frac{3\pi}{4}$ Q2

$\sin\theta = \frac{\sqrt{2}}{2}$ $\csc\theta = \sqrt{2}$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\cot\theta = -1$

B) 300°

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\csc\theta = -\frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{1}{2}$ $\sec\theta = 2$

$\tan\theta = -\sqrt{3}$ $\cot\theta = -\frac{\sqrt{3}}{3}$

C) $-\frac{5\pi}{6}$

$\sin\theta = -1/2$ $\csc\theta = -2$

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\sec\theta = -\frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$ $\cot\theta = \sqrt{3}$

D) $\frac{2\pi}{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = -\frac{1}{2}$ $\sec\theta = -2$

$\tan\theta = -\sqrt{3}$ $\cot\theta = -\frac{\sqrt{3}}{3}$

E) $\frac{13\pi}{3} - 6\frac{\pi}{3} - 6\frac{\pi}{3} = \frac{\pi}{3}$

$\sin\theta = \frac{\sqrt{3}}{2}$ $\csc\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = \frac{1}{2}$ $\sec\theta = 2$

$\tan\theta = \sqrt{3}$ $\cot\theta = \frac{\sqrt{3}}{3}$

F) $-240°$
$+360$
120

$\sin\theta = \sqrt{3}/2$ $\csc\theta = \frac{2\sqrt{3}}{3}$

$\cos\theta = -1/2$ $\sec\theta = -2$

$\tan\theta = -\sqrt{3}$ $\cot\theta = -\frac{\sqrt{3}}{3}$

G) $-\frac{7\pi}{2} + 4\frac{\pi}{2} + 4\frac{\pi}{2} = \frac{\pi}{2}$

$\sin\theta = 1$ $\csc\theta = 1$

$\cos\theta = 0$ $\sec\theta =$ UND

$\tan\theta =$ UND $\cot\theta = 0$

H) $135°$

$\sin\theta = \sqrt{2}/2$ $\csc\theta = \sqrt{2}$

$\cos\theta = -\sqrt{2}/2$ $\sec\theta = -\sqrt{2}$

$\tan\theta = -1$ $\cot\theta = -1$

I) $\frac{13\pi}{6} - 12\frac{\pi}{6} = \frac{\pi}{6}$

$\sin\theta = \frac{1}{2}$ $\csc\theta = 2$

$\cos\theta = \sqrt{3}/2$ $\sec\theta = \frac{2\sqrt{3}}{3}$

$\tan\theta = \frac{\sqrt{3}}{3}$ $\cot\theta = \sqrt{3}$

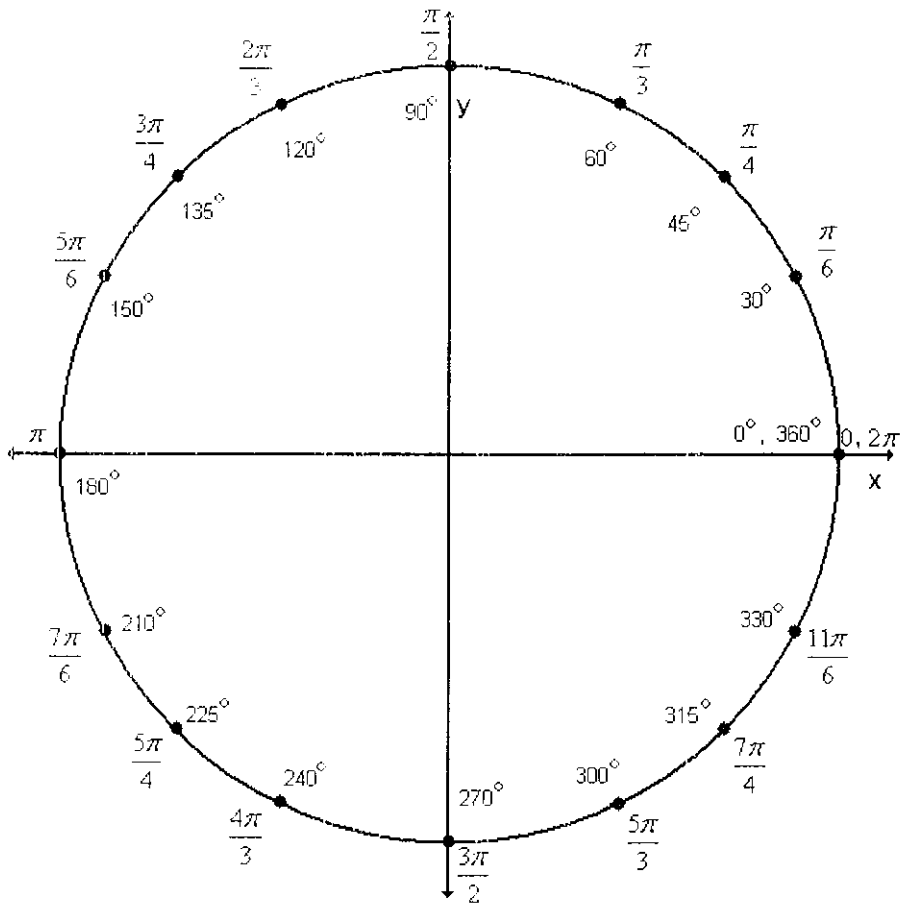
J) $-\frac{2\pi}{3} + 6\frac{\pi}{3} = 4\frac{\pi}{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\csc\theta = -\frac{2\sqrt{3}}{3}$

$\cos\theta = -\frac{1}{2}$ $\sec\theta = -2$

$\tan\theta = \sqrt{3}$ $\cot\theta = \frac{\sqrt{3}}{3}$

UNIT 6 WORKSHEET 7
USING THE UNIT CIRCLE



Use the unit circle above to find the <u>exact</u> value of each of the following. (Exact value means no decimal approximations.)

A) $\tan\frac{\pi}{4} =$ 1

B) $\cos\frac{2\pi}{3} =$ $-\frac{1}{2}$

C) $\cos\pi =$ -1

$-\frac{2\pi}{3} \times \frac{180^\circ}{\pi} = -120^\circ$
$+360^\circ$
240°

D) $\sin\frac{11\pi}{6} =$ $-\frac{\sqrt{3}}{3}$

E) $\tan\left(-\frac{2\pi}{3}\right) =$ $\sqrt{3}$

F) $\csc\frac{\pi}{3} =$ $\frac{2\sqrt{3}}{3}$

G) $\sec\frac{4\pi}{3} =$ -2

H) $\cos\left(-\frac{11\pi}{6}\right) =$ $\frac{\sqrt{3}}{2}$

I) $\sin\frac{13\pi}{4} =$ $-\frac{\sqrt{2}}{2}$

$-\frac{8\pi}{4}$

$\frac{5\pi}{4}$

J) $\csc\left(-\frac{5\pi}{6}\right) =$ -2

K) $\tan\left(-\frac{\pi}{6}\right) =$ $-\frac{\sqrt{3}}{3}$

L) $\cot\frac{2\pi}{3} =$ $-\frac{\sqrt{3}}{3}$

M) $\sec\left(-\frac{19\pi}{3}\right) =$ 2

$+\frac{6\pi}{3} + \frac{6\pi}{3} + \frac{6\pi}{3} + \frac{6\pi}{3}$

N) $\cot\frac{\pi}{4} =$ 1

O) $\cot\frac{11\pi}{6} =$ $-\sqrt{3}$

P) $\cos\left(-\frac{9\pi}{2}\right) =$ 0

$+\frac{4\pi}{2}$

$+\frac{4\pi}{2}$

Q) $\sin\frac{21\pi}{4} =$ $-\frac{\sqrt{2}}{2}$

$-\frac{8\pi}{4}$

$-\frac{8\pi}{4}$

R) $\cot\frac{7\pi}{4} =$ -1

S) $\sin\left(-\frac{7\pi}{6}\right) =$ $-\frac{\sqrt{3}}{2}$

T) $\cot\frac{26\pi}{3} =$ $-\frac{\sqrt{3}}{3}$

$-6\pi/3$

$-6\pi/3$

$-6\pi/3$ $-6\pi/3$

U) $\cos\frac{\pi}{3} =$ $\frac{1}{2}$

V) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

↑ radian form

$\sin\theta = -\frac{\sqrt{3}}{2}$ $\theta =$ $\frac{4\pi}{3}, \frac{5\pi}{3}$

W) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\csc\theta = \sqrt{2}$ $\theta =$ $\frac{\pi}{4}, \frac{3\pi}{4}$

X) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\tan\theta = \sqrt{3}$ $\theta =$ $\frac{\pi}{3}, \frac{4\pi}{3}$

Y) Find all angles θ in the interval $[0, 2\pi)$ that satisfy the expression:

$\sec\theta = undefined$ $\theta =$ $\frac{\pi}{2}, \frac{3\pi}{2}$

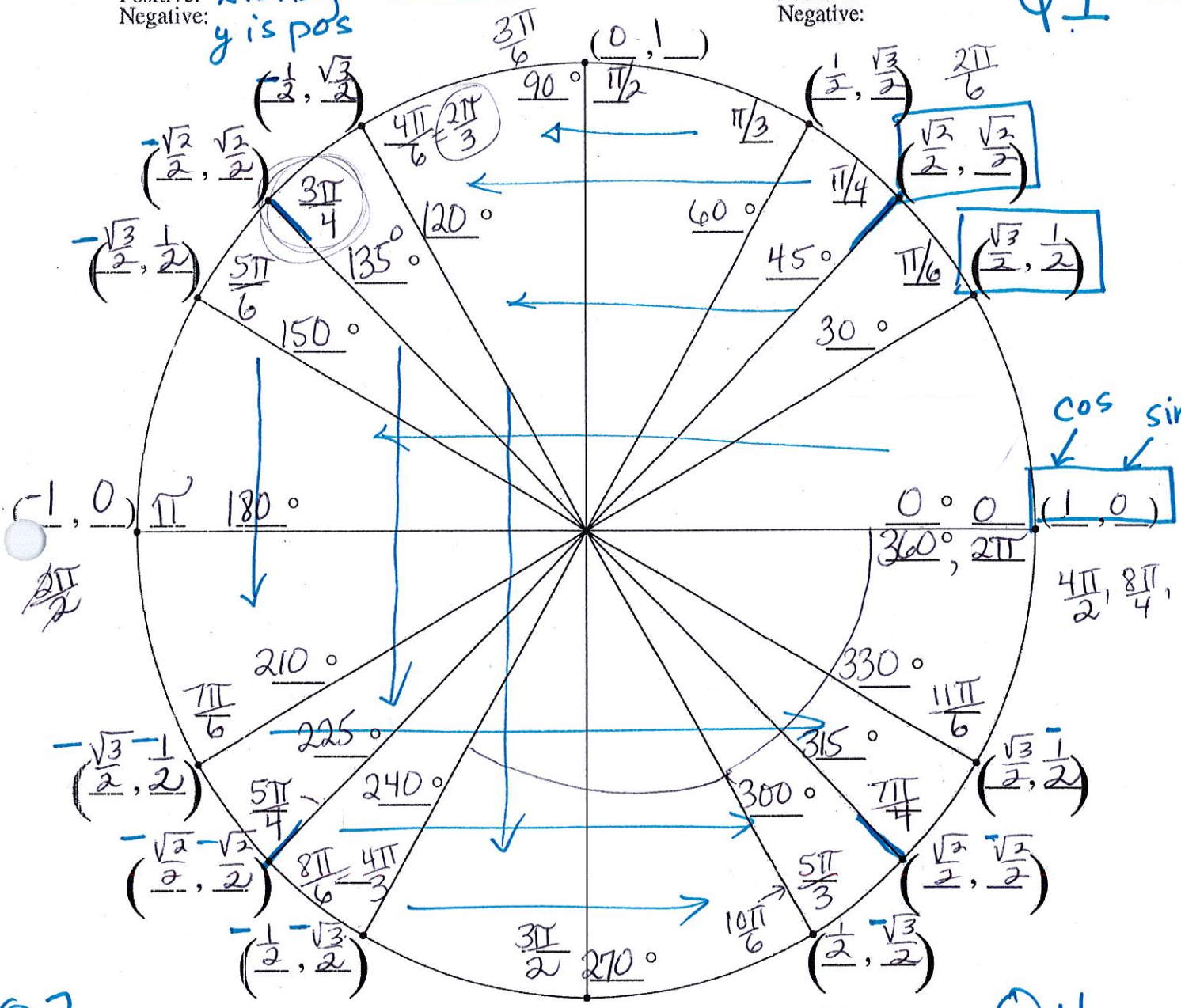
sin is positive, csc is positive

Study

Fill in The Unit Circle

Always all functions are positive

$x = \cos,\ y = \sin$



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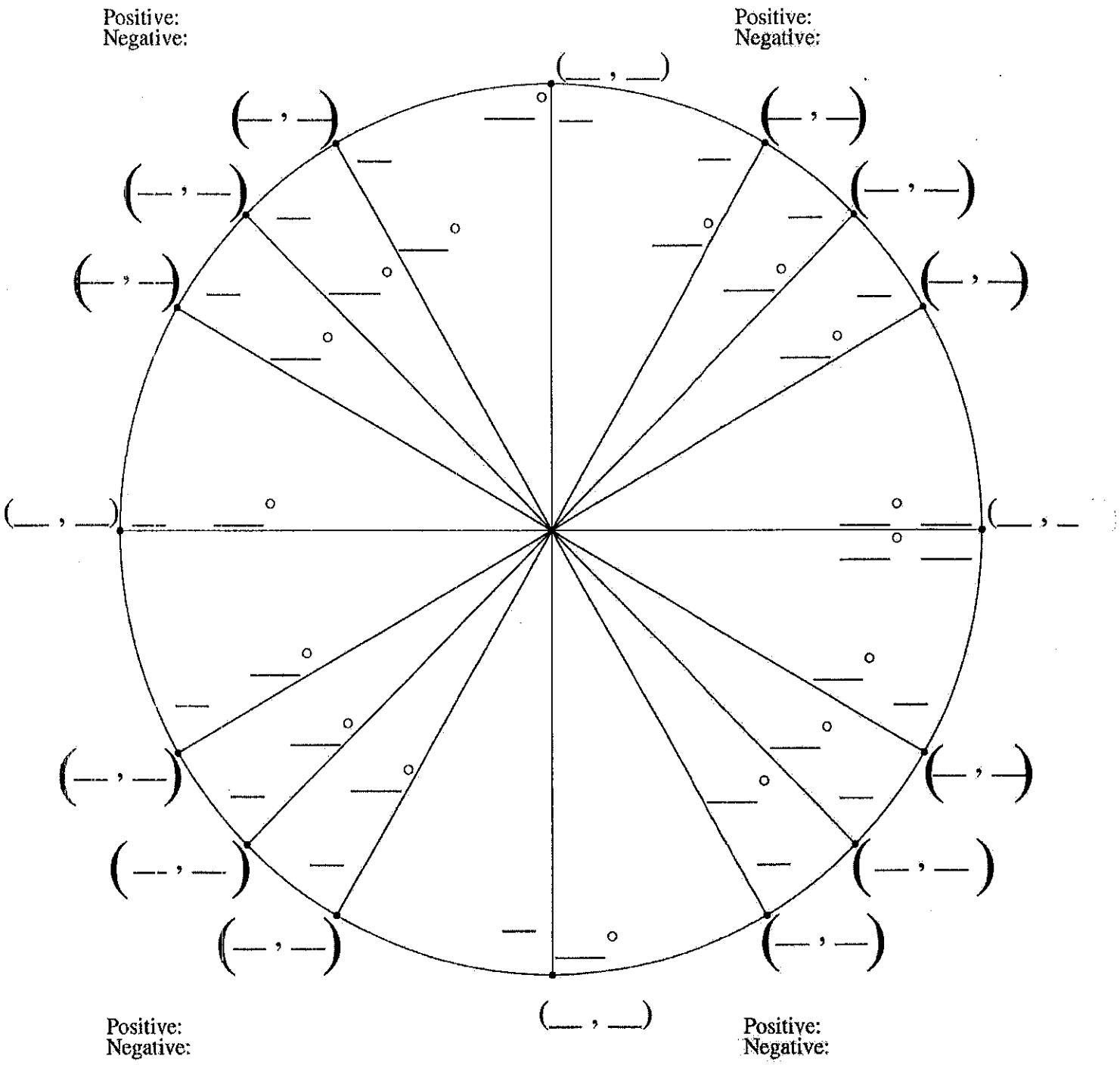
Trig

tan is positive; cot is positive

Carefully

cosine + sec are positive

Fill in The Unit Circle



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