Vectors Quiz (Part Two) Review

Find the dot product of u and v.

1. $u = 8i - 3j$, $v = -4i + 7j$

$(8 \cdot -4) + (-3 \cdot 7)$

$-32 + -21$

-53

Use the vectors $u = \langle -5, -8 \rangle$ and $v = \langle 6, 4 \rangle$ to find the indicated quantity.

2. $2u \bullet v$

$2\langle -5, -8 \rangle$

$\langle -10, -16 \rangle \bullet \langle 6, 4 \rangle$

$(-10 \cdot 6) + (-16 \cdot 4)$

$-60 + -64$

-124

3. $(u \bullet v)v$

$(-5 \cdot 6) + (-8 \cdot 4)$

$-30 + -32$

$-62 \langle 6, 4 \rangle \rightarrow \langle -372, -248 \rangle$

Find the angle between u and v.

Angle between vectors

$\theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\| \|v\|}\right)$

4. $u = \langle 7, -3 \rangle$, $v = \langle -11, 6 \rangle$

$\sqrt{7^2 + 3^2}$ $\sqrt{11^2 + 6^2}$

$u \cdot v = (7 \cdot 11) + (-3 \cdot 6)$

$(-77) + (-18)$

-95

$\theta = \cos^{-1}\left(\frac{-95}{\sqrt{58} \cdot \sqrt{157}}\right) \approx 174.6°$

5. $u = 2i + j$, $v = i - 2j$

$\langle 2, 1 \rangle$ and $\langle 1, -2 \rangle$

$(2 \cdot 1) + (1 \cdot -2)$

$2 + -2$

0

because the dot product equals 0, we know that the angle is 90°

6. $u = \langle 20, -4 \rangle$ $v = \langle 1, 12 \rangle$

$\sqrt{20^2 + 4^2}$ $\sqrt{1^2 + 12^2}$

$(20 \cdot 1) + (-4 \cdot 12)$

$20 + -48$

-28

$\theta = \cos^{-1}\left(\frac{-28}{\sqrt{416} \cdot \sqrt{145}}\right) \approx 96.5°$

Decide whether the vector is orthogonal, parallel or neither.

7. $u = \langle 5, 7 \rangle$, $v = \langle -15, -21 \rangle$

$u \cdot v = (5 \cdot -15) + (7 \cdot -21)$

$-75 + -147$

-222

the dot product $\neq 0$

$\therefore$ these vectors are not orthogonal

Compare their slopes

CROSS-MULTIPLY

$\frac{7}{5} \stackrel{?}{=} \frac{-21}{-15}$

$7(-15) \stackrel{?}{=} 5(-21)$

$-105 = -105$ ✓

They are equal

$\therefore$ Parallel

8. $u = \left\langle \frac{-2}{3}, \frac{3}{4} \right\rangle$
$v = \langle 9, 8 \rangle$

$u \cdot v = \left(-\frac{2}{3} \cdot 9\right) + \left(\frac{3}{4} \cdot 8\right)$
$-\frac{18}{3} + \frac{24}{4}$
$-6 + 6$
0

these vectors are orthogonal

9. $u = \langle -1, -4 \rangle$
$v = \langle 3, 6 \rangle$

$u \cdot v = (-1 \cdot 3) + (-4 \cdot 6)$
$-3 + -24$
-27
not orthogonal

Compare their slopes:

$\frac{-4}{-1} \stackrel{?}{=} \frac{6}{3}$

$-12 = -6$
NO!

they are neither parallel, nor orthogonal

Find $proj_v u$ and write u as the sum of two vector components.

10. $u = \langle 2, -4 \rangle, v = \langle -5, -3 \rangle$

Step 1: Find $u \cdot v$
$(2 \cdot -5) + (-4 \cdot -3)$
$-10 + 12$
2

Step 2: Find $\|v\|^2$ (like magnitude without $\sqrt{\ }$)
$5^2 + 3^2$
$25 + 9$
34

Step 3: Find w_1
$w_1 = \frac{2}{34} \langle -5, -3 \rangle$
$w_1 = \left\langle -\frac{5}{17}, -\frac{3}{17} \right\rangle$

Step 4: Find w_2 $(u - w_1)$
$w_2 = \langle 2, -4 \rangle - \left\langle -\frac{5}{17}, -\frac{3}{17} \right\rangle$
$w_2 = \left\langle \frac{39}{17}, -\frac{65}{17} \right\rangle$

11. $u = \langle 1, -5 \rangle, v = \langle 0, 6 \rangle$

Step 1: $(1 \cdot 0) + (-5 \cdot 6)$
$0 + -30$
-30

Step 2: $0^2 + 6^2$
36

Step 3:
$w_1 = \frac{-30}{36} \langle 0, 6 \rangle$
$w_1 = \langle 0, -5 \rangle$

Step 4: $w_2 = u - w_1$
$\langle 1, -5 \rangle - \langle 0, -5 \rangle$
$w_2 = \langle 1, 0 \rangle$

Find the work done for each problem.

12. Carl is carrying a pack of books from the table to the shelf with a force of 40 pounds at a 70° angle. If the distance from the table to the shelf is 160 feet horizontally. How much work has Carl done when he carries these books?

FORCE $= \langle 40 \cos 70°, 40 \sin 70° \rangle$
$\langle 13.7, 37.6 \rangle$

DISPLACEMENT: $\langle 160, 0 \rangle$

WORK = FORCE $\cdot$ DISPLACEMENT
$(13.7 \times 160) + (37.6 \times 0)$
$2192 + 0$
2192 pounds/ft

13. How much work is done by a force **F** = <4,7> in moving an object from (-5, 6) to (2, 9)?

WORK = force $\cdot$ displacement
$\langle 4, 7 \rangle \cdot \langle 7, 3 \rangle$
$(4 \cdot 7) + (7 \cdot 3)$
$28 + 21$
59

initial ↑ terminal

position vector: $\langle 2 - -5, 9 - 6 \rangle$
$\langle 7, 3 \rangle$